Phase Transitions of 2D Josephson Tunnel Junction Arrays

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Phase Transitions of 2D Josephson Tunnel Junction Arrays

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We have studied square 2D arrays of Josephson tunnel junctions at different values of the normalized magnetic flux per cell f. At f=0 Kosterlitz-Thouless behaviour is observed in the linear as well as in the non-linear resistance. For low nonzero values of f the linear resistance shows thermally activated behaviour at low temperatures with f-dependent activation energy. At f=1/2 the phase transition is different from that at arbitrary or zero field

We have fabricated and studied square 2D arrays of superconducting islands, weakly coupled with Josephson junctions. Such arrays can on one hand be viewed as model systems representing certain aspects of granular films, on the other hand as physical representations of the two-dimensional X-Y model. In the latter Kosterlitz-Thouless (K-T) transitions are expected to occur [1]. In these arrays however, the coupling energy between two superconducting islands connected with a Josephson junction, is temperature dependent. In order to compare experiment with theory, a normalized temperature τ has to be defined as the quotient of the thermal and the Josephson coupling energy:

$$\tau = \frac{k_B T}{(\hbar/2e)\, i_c(T)} \quad , \tag{1}$$

where $i_c(T)$ is the critical current per junction. In zero field, the K-T theory predicts a square-root cusp behaviour in the linear resistance near the K-T transition temperature and a universal jump in the exponent of the non-linear resistance[2,3].

In our group, we fabricate square arrays of junctions: every superconducting island is connected with four neighbours. The junctions are niobium oxide tunnel junctions and are made using a shadow evaporation method. The junctions measure 1 x 0.2 μm. A K-T transition temperature of about 5 K is chosen by variation of the oxide thickness. At this temperature the critical current per junction is typically 200 nA. With these values for the transition temperature and the critical current very detailed measurements are possible.

To use the τ scale as the relevant temperature scale, the critical current per junction as a function of temperature has to be known. In our arrays this critical current is determined from the array I-V characteristics. We previously reported on phase transitions in arrays, where the exponent of the non-linear resistance clearly reflected K-T behaviour [4]. However, the linear resistance failed to show the predicted square root cusp temperature dependence. For the fully frustrated array a different transition occurred with K-T like aspects.

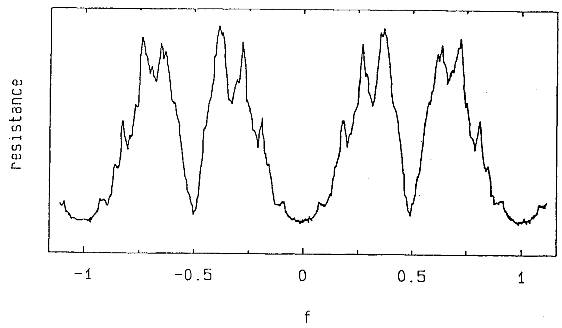

Figure 1, The linear resistance as a function of the applied magnetic field for an array of 16 islands wide and 128 long. At f=0 the resistance is about zero.

Recently our fabrication procedure was improved, probably resulting in more identical junctions. In figure 1 a typical magneto-resistance curve at low temperatures is given. Along the x-axis the flux per cell normalized to the flux quantum (frustration) is plotted. For f=1/2 (full frustration) the resistance exhibits a very pronounced dip. Clear dips can also be seen at f=1/3,2/3, f=1/5,4/5 and f=1/10,9/10. The resistance is periodic with one flux quantum per cell. This periodicity is found until f=100 in these arrays. The plots are symmetric around f=0. This periodicity and symmetry reflect the fundamental properties of the ground state for arrays in the presence of a perpendicular field.

For an array of 128 islands wide and 384 long, the linear resistance for several values of f is given in figure 2. The dashed line is a fit to the formula expected for the resistive, square-root cusp behaviour [2] :

$$\frac{R}{R_n} = a \exp\{ -b / (\tau - \tau_c)^{0.5} \} \quad . \tag{2}$$

with parameters a=1.6, b=4.0 and τ_c=0.95. In the low resistance regime, the experimental curve deviates from this theoretically predicted behaviour.

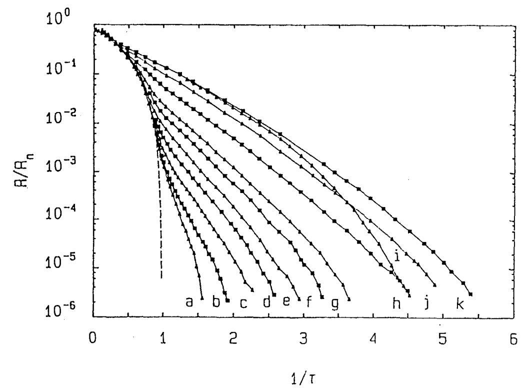

Figure 2, The normalized resistance as a function of the inverse normalized temperature for several values of the applied field. a: f=0, b: f=0.0008, c: f=0.002, d: f=0.0038, e: f=0.0068,f: f=0.014, g: f=0.025, h: f=0.078, i: f=0.5, j: f=0.19, k: f=0.4.

The curve *i* in figure 2 is the one for the f=1/2 case. It is obvious that f=1/2 is a special, more ordered state than other values of f. At $1/\tau \approx 4.5$, the curve for f=0.08 is even higher. Here the resistance is 20 times smaller than the maximum resistance at the same temperature. The dip at f=1/2 is very pronounced, as can also be seen in the magneto-resistance curve of figure 1. For f smaller than 0.08 the curves of figure 2 decrease approximately exponentially with inverse normalized temperature. This dependence can be expressed as:

$$\ln (R/R_n) = -\alpha \tau^{-1} \qquad (3)$$

The slopes α vary continuously from 9 at f=0 to about 2.5 at f=0.08.

In analogy with the dirty thin film analysis, one can calculate the energy of a single vortex in a long array of N cells wide. This energy in terms of the coupling energy as a function of the distance y from the centre is given by:

$$U_1 = \pi \ln \{ \frac{2N}{\pi} \cos(\pi y/N)\} + U_c \ , \qquad (4)$$

where U_c is the core energy of the vortex. A small magnetic field will also give a contribution to the energy of a vortex in the array. This energy for a vortex of the most favourable sign, again in terms of the coupling energy and as a function of the distance from the centre, is given by:

$$U_2 = -\frac{\pi^2 N^2}{2} f \{ 1 - 4 (y/N)^2 \} \quad . \qquad (5)$$

If $U_1+U_2 < 0$, the system can lower its ground state energy by introducing vortices in the array. As long as $U_1+U_2 > 0$, only thermally activated vortices will be present. In that case, using the expressions (4) and (5), it is possible to calculate the potential barrier as a function of the frustration.

With the estimated residual field and a cell size of 25 μm^2, we expect that the smallest average practical value of f for our arrangement is of the order of 5×10^{-4}. For N=128, this field leads to a value of U_2 which is larger than U_1. Vortices will therefore be present in the array and the potential barrier will be considerably reduced. This explains why thermally activated behaviour with a slope of 9 can be observed for 'zero' field.

To verify the previous concept, we have fabricated an array of 16 islands wide and 128 islands long in which there is a practically accessible range of f values where $U_1+U_2 > 0$. The linear resistance shows a qualitatively similar picture to figure 2. The experimental slopes in terms of the coupling energy E_J are plotted as a function of f in figure 3. These data can be compared with predictions derived from the analytical expressions (4) and (5) as well as with numerical model calculations for a discrete array of size N. The results are indicated in figure 3 as well. The analytical calculation yields the same values for the barrier as the model calculation if for U_c a value of 4.5 is used. Above f=0.009, the computer model introduces vortices in the ground state of the array. For larger values of f the potential barrier is therefore unknown.

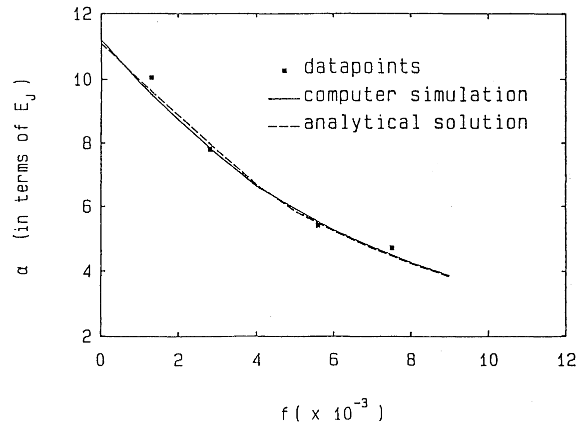

Figure 3, The energy required for a vortex to cross the array as a function of the frustration.

We conclude that the experimental data are in excellent agreement with the model predictions for N=16. We are convinced that the linear resistance of the larger arrays is also strongly modified in an analogous way by a residual magnetic field. The field does not primarily lead to a flux flow addition to the resistance, but to a modification of thermally activated vortex crossing.

ACKNOWLEDGEMENT

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